TEMPERATURE VARIATION ALONG A CYLINDRICAL HEAT BRIDGE IN CRYOGENIC PIPELINES

Yu. V. Bol'shakov, Ya. L. Brailovskii, and F. M. Pozvonkov

UDC 536.24

The effect of heat leakage to the outside surface of a heat bridge on the temperature variation along such a bridge has been analyzed qualitatively and quantitatively.

A performance analysis of cryogenic systems shows that much of the loss of cryogenic liquid in storage and transport is due to heat leaking in along heat bridges ("pin" connectors, temperature, pressure, or continuity gages, etc.).

The authors have made an attempt to analyze, qualitatively and quantitatively, the effect which heat leakage to the outside surface of a cylindrical joint such as a connector element, for instance, has on the temperature variation along it and on the magnitude of a thermal influx to the cryogenic liquid at the cold end in the cooling mode.

The problem is formulated as follows. There is a hollow cylinder of finite length R and wall thickness δ. Prior to the beginning of the process, the cylinder is in thermal equilibrium with the ambient medium at a temperature T_0. The temperature at one end of the cylinder becomes $T_c < T_0$ at time zero and is then maintained at this level throughout the cooling process. Heat leakage toward the outside surface of the cylinder occurs according to Fourier's law (heat leakage due to conduction through the insulation around the cylinder) and it is represented in the differential equation as a distributed heat source, a function of x and τ. The inside surface of the cylinder is ideally insulated.

If $R \gg \delta$ and $\lambda \gg \lambda_1$, then the temperature drop across the cylinder wall may be assumed equal to zero and the problem reduced to a one-dimensional one. Let the origin of coordinates be located at the hot end of the cylinder. Assuming that the thermophysical properties of the cylinder material and of the insulation material remain constant over the temperature range from T_0 to T_c, we have

$$c\gamma \frac{\partial T(x,\tau)}{\partial \tau} = \lambda \frac{\partial^2 T(x,\tau)}{\partial x^2} + W_1, \qquad \tau > 0,\ 0 \leqslant x \leqslant R, \tag{1}$$

where

$$W_1 = -\frac{\lambda_1}{\lambda \delta_1 \delta}[T(x,\tau) - T_0], \tag{2}$$

The initial and the boundary conditions are

$$\begin{aligned} T(x,0) &= T_0 = \text{const}, \\ T(0,\tau) &= T_0 = \text{const}, \\ T(R,\tau) &= T_c = \text{const}. \end{aligned} \tag{3}$$

A Laplace transformation [1] yields a solution to (1) in the form

$$\theta = \frac{\operatorname{sh}\left(\sqrt{\frac{\lambda_1}{\lambda\delta\delta_1}}\, x\right)}{\operatorname{sh}\left(\sqrt{\frac{\lambda_1}{\lambda\delta\delta_1}}\, R\right)} - \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2\mu_n \sin \mu_n \frac{x}{R}}{\mu_n^2 + \frac{\lambda_1}{\lambda\delta\delta_1} R^2} \exp\left[-\left(\mu_n^2 + \frac{\lambda_1}{\lambda\delta\delta_1} R^2\right) \mathrm{Fo}\right], \tag{4}$$

where $\mu_n = \pi n$.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 26, No. 6, pp. 1062-1066, June, 1974. Original article submitted February 1, 1974.

The thermal flux along the cylinder axis at the cold end is

$$q_\lambda = -\lambda \frac{dT_\lambda(R, \tau)}{dx} S. \tag{5}$$

For the steady-state condition (at $\tau \to \infty$) we have a solution

$$\theta = \frac{\operatorname{sh}\left(\sqrt{\frac{\lambda_1}{\lambda\delta\delta_1}}\, x\right)}{\operatorname{sh}\left(\sqrt{\frac{\lambda_1}{\lambda\delta\delta_1}}\, R\right)} \tag{6}$$

analogous to that obtained by the authors of [2] for the case of a temperature variation along the neck of a cryogenic container, as a function of the heat leaking in through the external insulation at a zero rate of gas flow.

With perfect thermal insulation around the cylinder ($\lambda_1 \to 0$) we have

$$\theta = \frac{x}{R} - \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{\mu_n} \sin \mu_n \frac{x}{R} \exp\left[-\mu_n^2 \mathrm{Fo}\right] \tag{7}$$

and the thermal flux along the cylinder at the cold end is

$$q_0 = -\lambda \frac{dT_0(R, \tau)}{dx} S. \tag{8}$$

For the steady-state condition we have

$$\theta = \frac{x}{R} \tag{9}$$

and

$$q = -\lambda \frac{T_c - T_0}{R} S. \tag{10}$$

A laboratory apparatus has been built for measuring the temperature variation along cylindrical heat bridges in cryogenic systems with various grades of insulation around their outside surfaces. It is a cryogenic pipe placed in a vertical position and consisting of two segments, upper and lower, coupled through a split connector. This connector, in turn, consists of two thin-walled coaxial cylinders (two heat bridges) welded on-end to the inner pipes and the outer jackets around those two segments. Generally, one seals the gap between both cylinders at the cold end by a bushing tightly slipped over the protruding ends of the inner pipes, and at the hot end by pressing a sleeve between the flanges of both outer jackets.

In the experiments we examined the temperature variation along the outer cylinder of this split connector, under vacuum in the gap between the cylinders and under thermal conditions corresponding to not more than 10^{-5} torr in the upper segment, ensuring an excellent thermal insulation for the inside surface of the outer cylinder. The thermal flux at the outside cylindrical surface was varied by adjusting the gas pressure inside the insulating vacuum cavity around the lower segment (the cold end of the split connector was located below the hot end).

The cryostat jacket was immersed in a thermostatic bath with water at a temperature of 9-11°C, while liquid nitrogen was poured into the inner pipe.

The temperature along the cylinder was measured with five copper–constantan thermocouples, the wires 0.1 mm in diameter brought out to a model ÉPP-09I automatically recording potentiometer. The second thermocouple junctions were placed in a Dewar flask with liquid nitrogen.

In order to analyze the thermal characteristics of such a split connector, a test compartment was set apart in the upper cryostat segment, as part of the inner pipe inside a guard container with liquid nitrogen which absorbed the heat leaking in laterally through the neck and from the outer jacket. This test compartment, together with the guard container, was located in the vacuum cavity of the upper segment. The level of liquid nitrogen was checked with a differential thermocouple whose junctions had been installed 80 mm apart along the height.

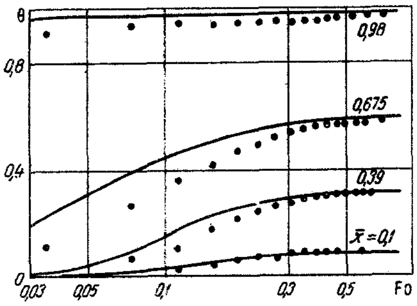

Fig. 1

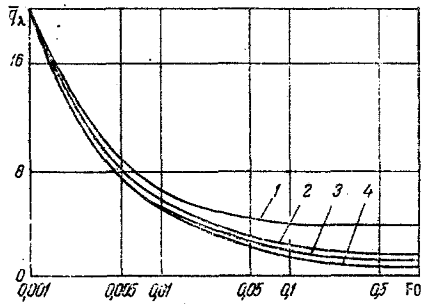

Fig. 2

Fig. 1. Temperature variation at several sections of the heat bridge during cooling ($\lambda_1 = 9.6 \cdot 10^{-3}$ W/m·deg). The curves represent Eq. (4); the dots indicate test values.

Fig. 2. Variation of the thermal flux at the cold end of the heat bridge during cooling: 1) $\lambda_1 = 8 \cdot 10^{-2}$ W/m·deg; 2) $1 \cdot 10^{-2}$; 3) $8 \cdot 10^{-3}$; 4) $1 \cdot 10^{-3}$.

The steady-state thermal influx to the inner cryostat pipe could be determined on the basis of the nitrogen flow rate, this nitrogen evaporating from the test compartment and measured with a model GSB-400 wet-type gas meter. The heat leakage toward the inner cryostat pipe under no-load conditions, determined without a heat bridge, amounted to 3.26 W at a gas pressure of 10^{-5} torr in the insulating cavity when hot.

The temperature variation along the cylindrical split connector was measured according to the following procedure. The pressure in the insulating vacuum cavity of the upper segment and in the gap between cylinders was reduced to 10^{-5} torr at most when hot. At the same time, water was poured into the thermostatic bath and the potentiometer was switched on. The gas pressure in the insulating cavity of the lower segment was set in three separate tests at 10^{-5}-10^{-6} torr, 1.0-1.5 torr, and 750 torr, respectively. The effective thermal conductivity of this tapered-vacuum insulation used in this experiment had been determined in a special cryostat and found to be $2.5 \cdot 10^{-4}$, $0.96 \cdot 10^{-2}$, and 0.15 W/m·deg, respectively. While the temperature was equalizing along the cylinder (the temperature difference between both ends of the bridge dropped to 2°C), liquid nitrogen was poured into the active pipe segment and into the guard container.

The temperature along the cylindrical connector was measured during the transient period of the cooling process as well as after a steady state had been reached. The transient time was not longer than 6-7 h and steady conditions were ascertained on the basis of constant temperatures at each cylinder section.

The results of the experiment are shown in Fig. 1, where the temperature measurements along the cylindrical heat bridge in the coling mode are compared with temperature calculations according to Eq. (4) for the same sections. These calculations were made on a model ÉVM-222 computer. The geometrical and the thermophysical parameters of the connector and of the insulation were as follows: $T_0 = 282$°K; $T_c = 77$°K; $R = 0.254$ m; $\delta = 1.5 \cdot 10^{-3}$ m; $\delta_1 = 19 \cdot 10^{-3}$ m; $\lambda = 11$ W/m·deg; and $\lambda_1 = 8.5 \cdot 10^{-3}$ W/m·deg.

The discrepancy between theoretical values and test data for the first stage of the cooling process can be explained by our inability to ensure an instantaneous cooling of the cold end of the heat bridge down to the boiling point of the cryogenic liquid.

Calculations and experiments performed for the purpose of analyzing the thermal characteristics of bridge connectors in practical designs of cryogenic systems have shown that, when heat is leaking toward the surface of cylindrical bridges through an insulation whose effective thermal conductivity is $\lambda_1 < 10^{-3}$ W/m·deg, the temperature variation along the heat bridge remains linear and that the amount of heat leaking toward the cryogenic liquids is determined by the thermal conductivity of the cylinder material as well as by its section area.

The relative thermal flux

$$\bar{q}_\lambda = \frac{q_\lambda}{q} \tag{11}$$

TABLE 1. Effect of Insulation Grade on the Amount of Heat Leaking along a Heat Bridge toward the Cryogenic Liquid

λ_1, W/m·deg	$1\cdot10^{-3}$	$2,8\cdot10^{-3}$	$7\cdot10^{-3}$	$1,1\cdot10^{-2}$	$1,4\cdot10^{-2}$
q_λ, W	2,45	3,06	3,8	4,3	4,66

at the cold end of the cylinder is shown in Fig. 2, as a function of time, based on calculations for various amounts of heat leakage through the insulation to the outside surface of this cylinder.

Calculations show that, at Fo < 0.01 and with $\lambda_1 < 1\cdot10^{-2}$ W/m·deg, the amount of heat leaking toward the cryogenic liquid while the heat bridge cools is determined by the specific heat of the cylinder material and remains almost independent of the insulation grade. At Fo > 0.01 and with $\lambda_1 > 10^{-3}$ W/m·deg, the amount of heat leaking toward the cold end of the cylinder increases as the thermal conductivity of the insulation λ_1 becomes higher.

The amount of heat leaking toward the cold end of the cylinder under steady-state conditions is shown in Table 1 for various grades of insulation around the outside surface of the heat bridge.

NOTATION

$T(x, \tau)$	is the temperature along a connector;
T_0	is the temperature at the hot end of a connector;
T_C	is the temperature at the cold end of a connector;
$\theta = (T_0 - T(x, \tau))/(T_0 - T_C)$;	
τ	is the time;
a	is the thermal diffusivity of the connector material;
c	is the specific heat of the connector material;
λ	is the thermal conductivity of the connector material;
γ	is the density of the connector material;
R	is the connector length;
δ	is the connector thickness;
λ_1	is the effective thermal conductivity of the insulation;
δ_1	is the effective thickness of the insulation around a heat bridge;
x	is the space coordinate;
$\bar{x} = x/R$;	
S	is the section area of a heat bridge;
$Fo = a\tau/R^2$	is the Fourier number.

LITERATURE CITED

1. A. V. Lykov, Theory of Heat Conduction [in Russian], Vysshaya Shkola, Moscow (1967).
2. S. P. Gorbachev and M. G. Kaganer, "Thermal flux at the neck of containers for storing cryogenic liquid," Trudy VNIIKriogenmash, No. 13 (1971).
3. M. P. Malkov et al., Textbook on Physicotechnical Principles of Deep Freezing [in Russian], GÉI, Moscow–Leningrad (1973).